Stability Margins and Low-Codimension Bifurcations of Indirect Filed Oriented Control of Induction Motor

Nizar Jabli[1,*], Hedi Khammari[2], Mohamed Faouzi Mimouni[3], Sultan Aljahdali[2]

[1]Department of Electronics, Higher institute of applied sciences and technology, Sousse, Tunisia
[2]Department of computer science, Faculty of computer science, Taief, Saudi Arabia
[3]Department of electrical engineering, National engineering school of, Monastir, Tunisia
*Corresponding author: jabli.nizar@yahoo.fr

Abstract The main purpose of this paper is to give a reasonably comprehensive discussion of what is commonly referred to as the bifurcation analysis applied to an indirect field oriented control of induction machines (IFOC). In the current work, we study the appearance of self-sustained oscillations in AC drives and compute their corresponding stability margins. As the dynamics is explored, a transition mode to chaotic states via codimension one Hopf bifurcations is detected. Based on qualitative approach, investigations of both parametric and phase plane singularities in IFOC induction motor lead to put into evidence equilibrium points and complex oscillatory phenomena such as limit cycles and chaotic behaviors. Furthermore we found out the bifurcation sets and the attraction basins related to such nonlinear phenomena. Bifurcations originated by system and control parameter fluctuations may lead to stability loss. The adequate remedy is to keep the parameters and the state variables inside the well known normal operating domains computed in this paper. It is worth noting that a rational use of the main analysis tools such as bifurcation sets and attraction basins permits to cancel non desired oscillations and limit cycles by choosing the appropriate initializations leading to the desired behavior. The interpretation of these results contributes to widen the understanding of the mechanism of certain types of singularities and the stability domain boundaries either in phase space or in parameter space and to demonstrate the suitability of bifurcation theory to solve stability problems in electric machines.

Keywords: attractors, multistability, attraction basins, bifurcation, chaos, induction machine

Cite This Article: Nizar Jabli, Hedi Khammari, Mohamed Faouzi Mimouni, and Sultan Aljahdali, "Stability Margins and Low-Codimension Bifurcations of Indirect Filed Oriented Control of Induction Motor." *International Transaction of Electrical and Computer Engineers System* 1, no. 1 (2013): 6-14. doi: 10.12691/iteces-1-1-2.

1. Introduction

Nonlinear dynamics may provide understanding and knowledge about systems which demonstrate complicated and irregular behavior. The general purpose of this paper is to identify generic bifurcations (as Saddle-node, Hopf and Bogdanov-Taken bifurcation, etc.) in an induction motor submitted to an Indirect Field-oriented control. Bifurcation is a dynamic behavior associated with loss of stability that can be caused by the errors in the estimate of the time constant. At the bifurcation point, existence and uniqueness of solutions is not guaranteed and a change in the number of solutions occurs.

Field oriented controllers (FOC) are frequently used as nonlinear controllers for induction machines, perform asymptotic linearization, and decoupling [1]. Stability of FOC is generally investigated regarding errors in the estimate of the rotor resistance. It has been previously shown that the speed control of induction motors through indirect Field-Oriented Control (IFOC) is globally asymptotically stable for any constant load torque. An analysis of saddle-node and Hopf bifurcations in IFOC drives due to errors in the estimate of the rotor time constant provides a guideline for setting the gains of PI speed controller in order to avoid Hopf bifurcation [2]. An appropriate setting of the PI speed loop controller permits to keep the bifurcations far enough from the operating conditions in the parameter space [3]. Recently, the qualitative methods became useful tools of analysis in the investigation of the power systems. The understanding of mechanism responses of such nonlinear dynamic system is based on the identification of both singularities of the phase plane (equilibrium, limit cycles, attraction basins, etc.) and singularities of the parameter plane (bifurcations, chaos, etc.) [4]. It has been proven the occurrence of either codimension one (saddle node and Hopf bifurcation) and codimension two bifurcation (Bogdanov-Takens or zero-Hopf bifurcation) in IFOC induction motors [5,6,7]. Other studies were concerned with the cancellation of sustained oscillations which are, in general undesirable.

Some of such studies proposed an 'oscillation killer' dedicated to adjust the system and control parameters so that one can get rid of limit cycles [8]. In [9], chaotic rotation can promote efficiency or improve dynamic characteristics of drives. Thus chaotic behavior, obtained

for some ranges of load torque and certain PI speed controller settings, is a desirable behavior in this case. An adequate combination between analytical and numerical tools may provide a deep understanding of some nontrivial dynamical behavior related to bifurcation phenomena in a self-sustained oscillator [10].

The robustness margins for IFOC of induction motors can be deduced from the analysis of the bifurcation structures identified in parameter plane [11]. Since the self-sustained oscillations in IFOC for induction motors may be due to the appearance of a Hopf bifurcation [12], an exhaustive study of the bifurcation structures is mainly devoted to preserve the local stability of the desired equilibrium point. The stability test given in [13] based on existence of quadratic Lyapunov functions may lead to a largest global asymptotic stability margins for IFOC induction motors. The robustness margins with respect to rotor time constant mismatches are obtained by iteratively applying the stability test for different PI settings.

The stabilizing effect of the harmonic injection revealed in [14], had led to a couple of advantages, namely torque enhancement and a greater robustness. The bifurcation analysis of a five-phase induction-motor drive submitted to a third harmonic injection allows undesirable nonlinear phenomena to be circumvented to some extent for a wide range of estimation errors. Power system stability analysis is the common framework of studies cited above. The accurate computation of stability margins lead to design the adequate controllers which are able to avoid undesirable behaviors and to bring the system to a stable steady state. In this paper we proposed some steps toward the development of stability analysis tools applied to IFOC induction motors.

In power systems, there is an immense need for exhaustive studies of parametric and phase plane singularities in order to assist the design of sufficient controllers. Thus we investigate the computation of attraction basins and the main features that may characterize the IFOC induction machines. Namely the transition modes to unwanted chaotic oscillations and the control bifurcations due mainly to controller's parameter variation.

Section 2 devotes to the equation model description of IFOC induction motor and some general reminders. Section 3 presents some features of multistability properties illustrated for both of equilibria and limit cycles. Section 4 provides further insight on the occurrence of some generic bifurcations of codimension one and two. A transition mode from Hopf bifurcation to chaotic orbits is described in section 5.

2. Plant Equation Description/General Remember

An autonomous system is generally described by a system of ordinary differential equations (ODEs) of the form:

$$\frac{dX}{dt} = f(X, \lambda) \; ; \; t \in IR, \; X \in IR^n, \; \lambda \in IR^p \qquad (1)$$

Where f is smooth. A bifurcation occurs at parameter $\lambda = \lambda_0$ if, crossing this value, the system behavior undergoes an abrupt change affecting the number and/or the stability of equilibria or periodic orbits of f .

As mentioned in previous papers [15], a two-parameter plane can be considered as made up of sheets (foliated representation), each one being associated with a well defined behavior such as a fixed point, or an equilibrium or a periodic orbit.

2.1. System Equations of Induction Machine

The equation model of indirect field-oriented control of induction motor can be described by the following 4th-order nonlinear autonomous system:

$$\begin{cases} \dot{x}_1 = -c_1.x_1 + (k.c_1 / u_2^0).x_2.x_4 + c_2.u_2^0 \\ \dot{x}_2 = -c_1.x_2 + (k.c_1 / u_2^0).x_2.x_4 + c_2.x_4 \\ \dot{x}_3 = -c_4.(c_5(x_1.x_4 - x_2.u_2^0) - T_L) \\ \dot{x}_4 = k_i.x_3 - k_p.c_4.(c_5(x_1.x_4 - x_2.u_2^0) - T_L) \end{cases} \qquad (2)$$

x_1, x_2 , x_3 and x_4 are the variable states, where:

x_1 and x_2 denote the direct and the quadratic component of the rotor flux, respectively. x_3 being the difference between reference and the real mechanical speed. x_4 present's the quadratic component of the stator current results from the outer loop PI.. k is the ratio of the rotor time constant τ_r to its estimate τ_e and u_2^0 is a design parameter. k_p and k_i are the proportional and the integral gains, respectively.

c_1, c_2 , c_3 and c_4 are constants, where:

$c_1 = \tau_r^{-1} = L_r.R_r^{-1}$ is the rotor flux time constant.

$c_2 = L_m.\tau_r^{-1}$.

$c_3 = f_c.\tau_r^{-1}$, f_c is the friction constant.

$c_4 = n_p.J^{-1}$.

2.2. General Remember

The parameterized nonlinear differential system (2) can present multiple equilibria as a single parameter varies. A local bifurcation at an equilibrium happens when some eigenvalues of the parameterized linear approximating differential equation cross some critical values such us the origin or the imaginary axis. Self-sustained oscillations in IFOC of induction motors can be originated by a codimension one bifurcation namely the Hopf bifurcation (H). Such kind of bifurcation can be computed from differential system (2), when a pair of complex conjugate eigenvalues among the eigenvalues set of the associate linearized system change from negative to positive real parts or vice versa. Therefore the Hopf bifurcation results from the transversal crossing of the imaginary axis by the pair of complex conjugate eigenvalues. Such bifurcation is said to be supercritical if the periodic branch is initially stable and subcritical if the periodic branch is initially unstable. The singularities of the phase plane are the solutions of 4th order autonomous differential system describing the IFOC induction motor (Equilibrium points, limit cycles, chaotic orbits...), each solution involves four eigenvalues describing its stability. A Saddle-node

bifurcation (or Fold), or a limit point (LP) is a codimension one bifurcation which occurs when a single eigenvalue is equal to zero.

Some codimension two bifurcation points are considered in this paper such as the cuspidal point (CP), the Bogdanov-Takens bifurcation (BT) and the Generalized Hopf bifurcation (GH).

In a two parameter plane, a Bogdanov-Takens bifurcation happens for the assumption of an algebraically double zero eigenvalue, therefore, in a (k, T_L)-plane, a Bogdanov-Takens (BT) bifurcation occurs when an equilibrium point has a zero eigenvalue of multiplicity two. In the neighborhood of such bifurcation point, the system has at most two equilibria (a saddle and a non saddle) and a limit cycle. The limit cycle results from a non saddle equilibrium which undergoes an Andronov-Hopf bifurcation. Numerically, the normal Lyapunov exponents calculated in the Hopf bifurcation point are negative which means that these periodic orbits are born stable [16]. The saddle and nonsaddle equilibrium collide and disappear via a saddle-node bifurcation. This cycle degenerates into an orbit homoclinic to the saddle and disappears via a saddle homoclinic bifurcation.

A generalized Hopf (GH) bifurcation or Bautin bifurcation appears when a critical equilibrium has a pair of purely imaginary eigenvalues. The singular curves of the parameter plane corresponding to codimension-1 bifurcations may contain singular points of higher codimension [4]. The simplest one located on a fold curve has the codimension-2, a fold cusp. It is the meeting point of two fold arcs. A Bogdanov-Takens bifurcation point (BT) will be identified on a saddle-node bifurcation curve, and a generalized Hopf bifurcation (GH) on a Hopf bifurcation curve.

3. Multistability in IFOC Induction Motor

Multistability is a major property of non linear dynamical systems and means the coexistence of more than one stable behaviour for the same parameters set and for different initial conditions. Solving the differential system equation (2), the trajectory in state space will head for some final attracting region, or regions, which might be a point, curve, area, and so forth. Such an object is called the attractor for the system. Really the nonunicity of these attractors led primarily to characterize each stable

state by a domain of stability or an attraction basin. These domains include one or more open sets of points in the phase space corresponding to all the initial conditions combinations for which the solutions of the system (2) converge towards such stable state. Thus, an attraction basin is a stability domain (D) of an attractive set (or attractor) having a border (F). The analysis of the properties of stability domain (D) of these attractors and its border (F) (Connectivity, complex shape, fractal…) is used in a lot of works as an important tool in studying the behaviour of a dynamic electrical circuit. We let's up consider a geometrical transformation T associated to the differential system (2). Theoretically, T can be a diffeomorphism (invertible) or an endomorphism (non unicity of T^{-1}. A basin of attraction is connex if the punctual transformation T is invertible. Whereas, in the case of a noninvertible transformation (or endomorphism) [17,18], the attraction basin can be made of a finite or infinite number of non connected areas, or a single connected area but bored by holes (basin multiply connected) [4].

Since the parameter space is foliated, two Saddle-node bifurcation curves continued from two successive limit point bifurcation are generally the boundaries of three different sheets (two stable sheets related through a third unstable one) [19]. This type of bifurcation feature exhibits phenomena of jump and hysteresis. Furthermore, the Double-Hopf bifurcation occurrence leads generally to the coexistence of two periodic solutions, also called limit cycles. Reciprocally, it is possible but not rigorously proven that the coexistence of a pair of equilibrium points or limit cycles under parameter variation is related to a limit point or to double-Hopf bifurcation appearance, respectively.

3.1. Multistability of Equilibrium Point

For the parameters $k = 4$, $k_p = 0.4$, $k_i = 1$ and $T_L = 0.5$ two different equilibrium points are identified :The first one is $(x_{10}^*, x_{20}^*, x_{30}^*, x_{40}^*) = (0.2764, -0.1383, 0, 1.309)$, solution of the differential system (2) for the initial conditions set $(x_{10}, x_{20}, x_{30}, x_{40}) = (1, 1, 0.1, 0.1)$ whereas $(x_{10}^*, x_{20}^*, x_{30}^*, x_{40}^*) = (0.7236, -0.3618, 0, 0.191)$ is the second one, and similarly a solution of (2) for the following initial conditions set $(x_{10}, x_{20}, x_{30}, x_{40}) = (1, -1, 0.1, 0.1)$.
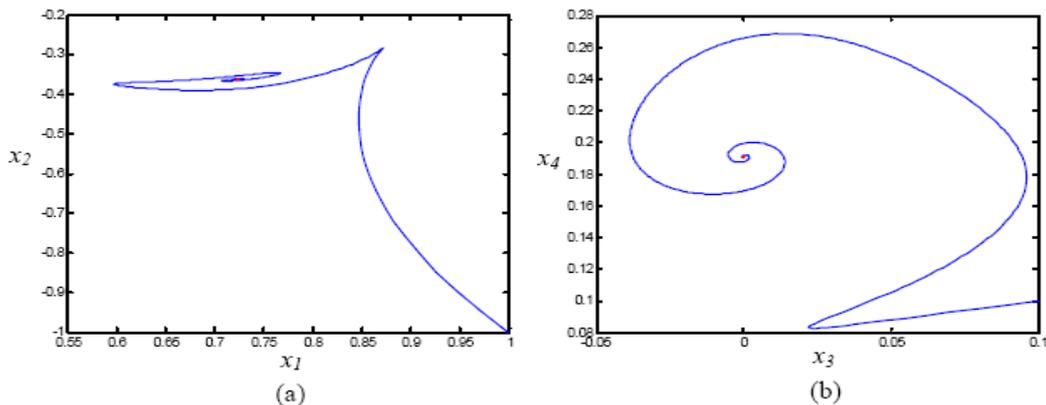

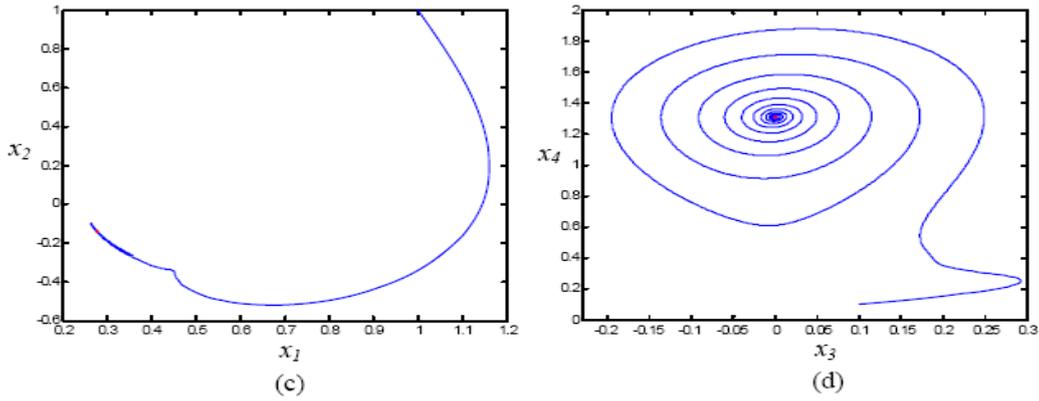**Figure 1.** Equilibrium Point EP1 in phase planes (*x1, x2*) and (*x3, x4*)

Figure 2. Equilibrium Point EP2 in phase planes ($x1$, $x2$) and ($x3$, $x4$)

The phase trajectories converging to equilibrium points are given in both of phase planes ($x1$, $x2$) and ($x3$, $x4$) see Figure 1 and Figure 2.

For more complete characterization of each of the equilibrium points one can compute, in phase planes (x_1, x_2) and (x_3, x_4), the stability domains which are the sets of initial conditions leading to one of such equilibria, such domains are called attraction basins and can be connected or not or fractal in some cases. The attraction basins of the two equilibrium points in phase planes (x_1, x_2) or (x_3, x_4) are given in Figure 3 and Figure 4 respectively. The knowledge of the initial conditions sets leading to such or such behaviour enables to maintain the state variable values in appropriate ranges so that we obtain always the desired system behaviour.
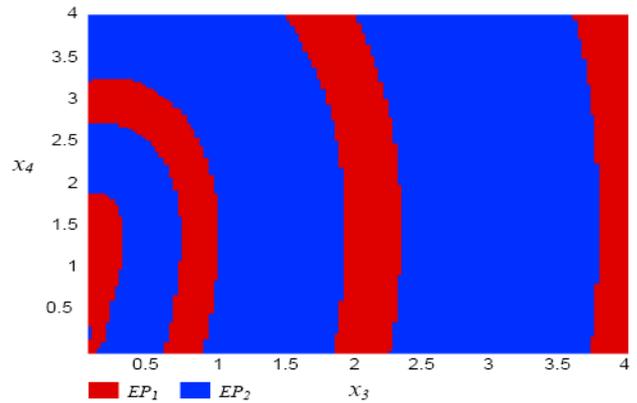
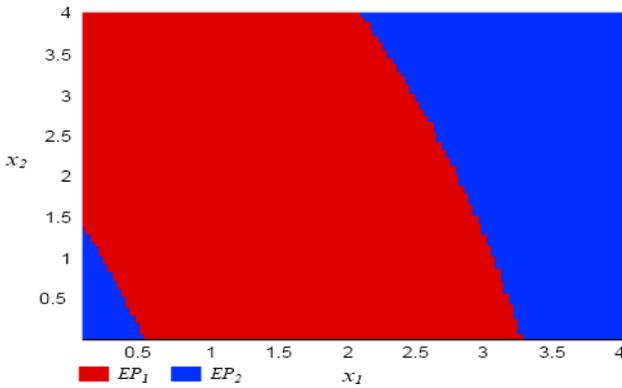**Figure 4.** Attraction basins of Equilibria EP1 and EP2 in ($x3$, $x4$)

According to the Figure 3 and Figure 4, the stability domains of the two equilibrium points are apparently connected and scrolled around each other, besides the attraction basin of the equilibrium point EP2 are larger than the EP1's one in phase planes (x_3, x_4)

3.2. Multistability of Limit Cycles

In Figure 5 both of the red and the blue limit cycles coexist for the parameters $k = 0.02017$; $k_p = 0.15$; $k_i = 1.01$ and $T_L = 10.1$, but for different initial conditions sets $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.19, 0.5, 0, 0)$ and $(x_{10}, x_{20}, x_{30}, x_{40}) = (2.2, 7.5, 0.5, 7.5)$ respectively.
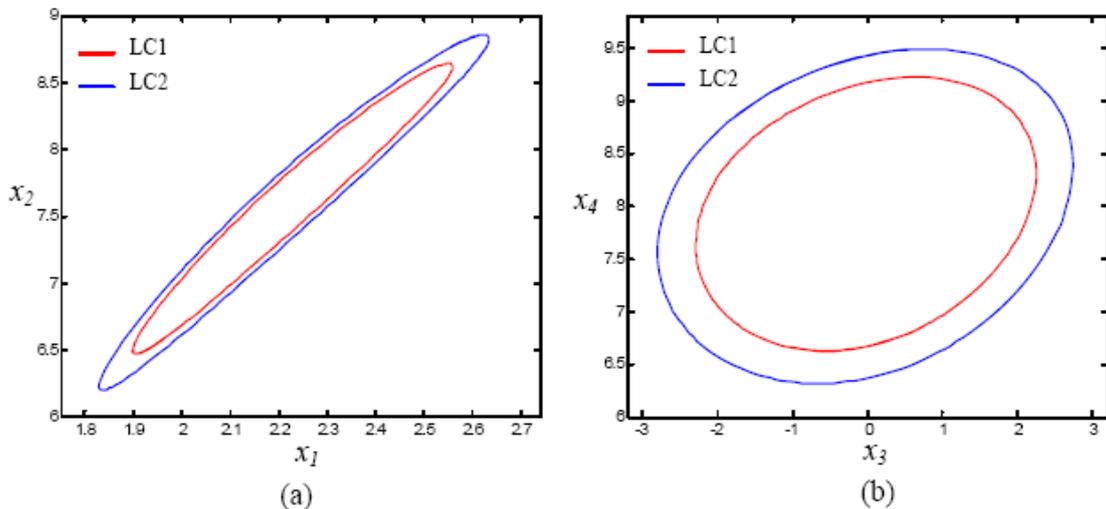

Figure 3. Attraction basins of Equilibria EP1 and EP2 in ($x1$, $x2$)

Figure 5. Limit cycles LC1 and LC2 in phase planes ($x1$, $x2$) and ($x3$, $x4$)

The computation of the two limit cycles attraction basins leads to determine two different regions (red and blue) in phase planes ($x1$, $x2$) and ($x3$, $x4$) see Figure 6 and Figure 7.
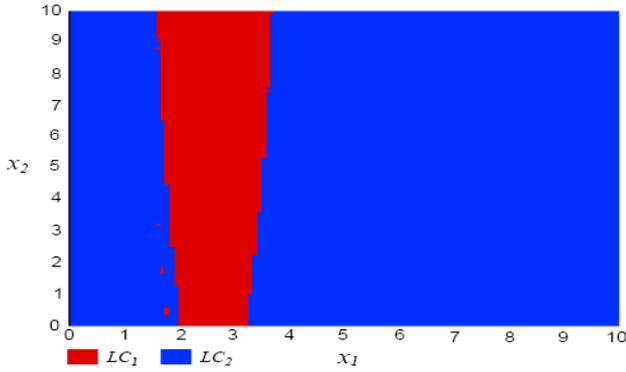
It is obvious to conclude that there is no other (third) behaviour else than the well defined limit cycles in the considered regions of the phase planes mentioned above.

4. Bifurcation Sets

4.1. Hopf Bifurcation Detection

For a load torque value $T_L = 10$, the phase trajectories undergo two important qualitative changes under the variation of the parameter k. For $k = 0.1$, Figure 8a presents a limit cycle illustrated by two closed trajectories in phase planes ($x1$, $x2$) and ($x3$, $x4$). The phase portrait in ($x1$, $x2$) presents an auto-intersection which disappears for $k = 0.17$ as shown in Figure 8b, this is mainly due to the state variables spectral composition change.

Then for $k = 0.18$ the limit cycle disappears and an equilibrium point occurs instead of it see Figure 8c. One can guess the existence of a Hopf bifurcation for $0.17 < k < 0.18$ which can be computed easily using an adequate continuation program.

The Hopf bifurcation phenomenon, being one of the possible reasons for the oscillatory behaviour, is an abrupt qualitative change that can be accompanied by a 'quantitative' change namely the spectral reorganization of the oscillating state variables. The spectral analysis of periodic solutions, by means of Fourier Transform, was employed in [15] to characterize a succession of saddle-node bifurcation in a parameter plane. Thus, the spectral approach applied to periodic solutions in non autonomous systems may be extended to limit cycles in autonomous case.
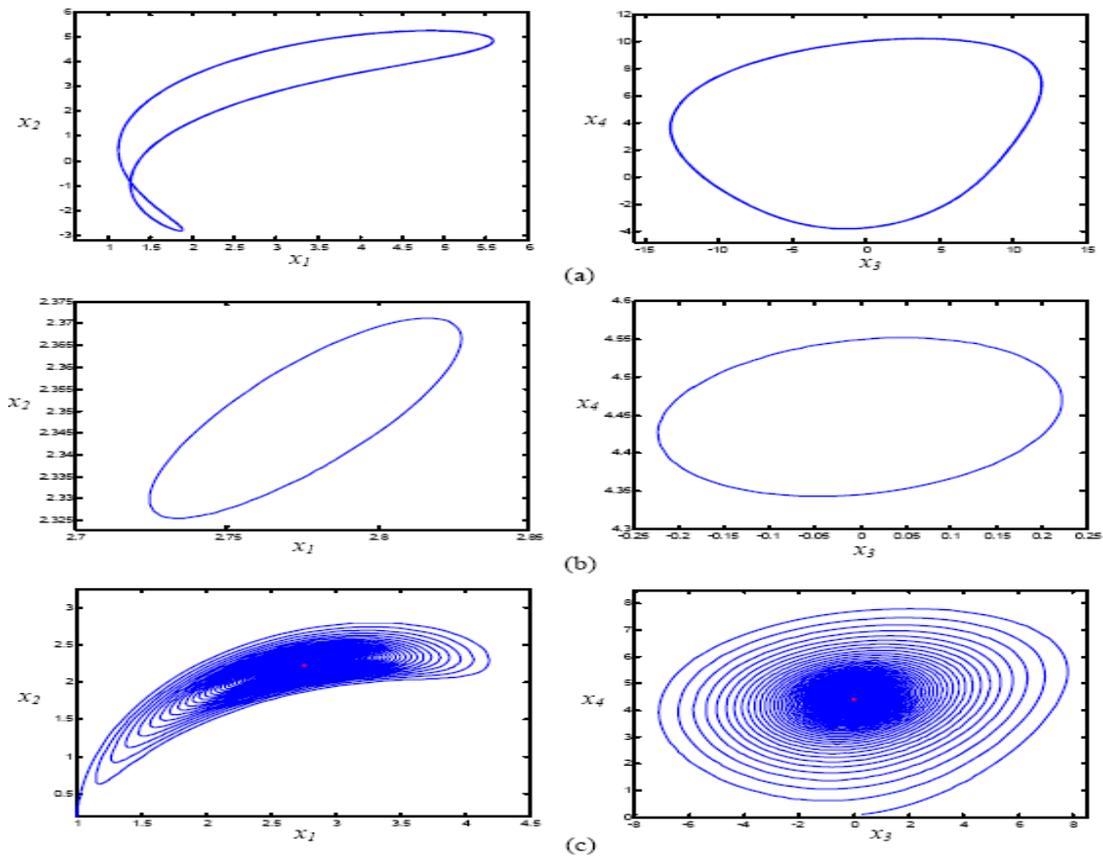

Figure 6. Attraction basins of Limit cycles LC1 and LC2 in ($x1$, $x2$)
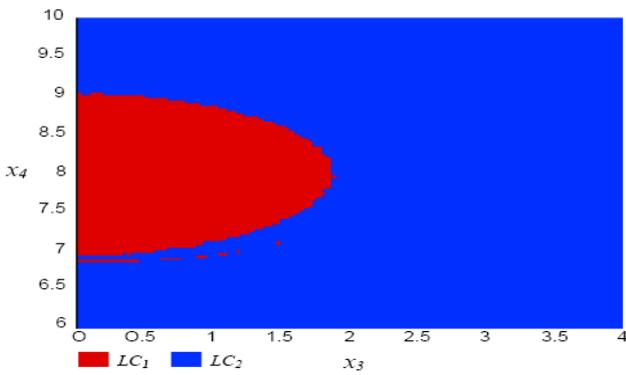
Figure 7. Attraction basins of Limit cycles LC1 and LC2 in ($x3$, $x4$)

Figure 8. Hopf Bifurcation: phase trajectories in phase planes ($x1$, $x2$) and ($x3$, $x4$). For $kp = 0.4$, $ki = 1$ and $TL = 0.5$. (a) $k = 0.1$, (b) $k = 0.17$, (c) $k = 0.18$

4.2. Limit Point and Hopf Bifurcation Point

Starting from a located initial equilibrium or a periodic orbit, numerical continuation is devoted to follow such special behaviour as a single active parameter varies. The starting point is an equilibrium point $(x_{10}^*, x_{20}^*, x_{30}^*, x_{40}^*) = (0.2764, -0.138, 0, 1.31)$ computed for the parameters $k = 4$, $k_p = .4$, $k_i = 1$ and $T_L = 0.5$, a continuation method permits to obtain the evolution of x_1 versus the values of k (see Figure 9).

Three singularities are obtained on such curve: two Hopf bifurcation points (Ns) and a limit point F (Saddle-node bifurcation or Fold). The Saddle-node bifurcation possesses has one of its eigenvalue equal to zero and the following coordinates in phase space: $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.34495, -0.21835, 0, 0.8165)$ and the corresponding eigenvalues are: $(-4.0652 +i12.01, -4.0652 - i12.0101, -0.0075, -4.73256e - 005)$. The two neutral saddles have the following coordinates: $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.35572, -0.23302, 0, 0.75052)$ with the associates eigenvalues: $(-4.07115+i11.0785, -4.06523-i11.0785, -0.17818,$

$0.17818)$ and $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.43023, -0.35261, 0, 0.34257)$ with the eigenvalues $(-4.08605+ i6.62942, -4.06523 - i6.62942, -0.57292, 0.57292)$.
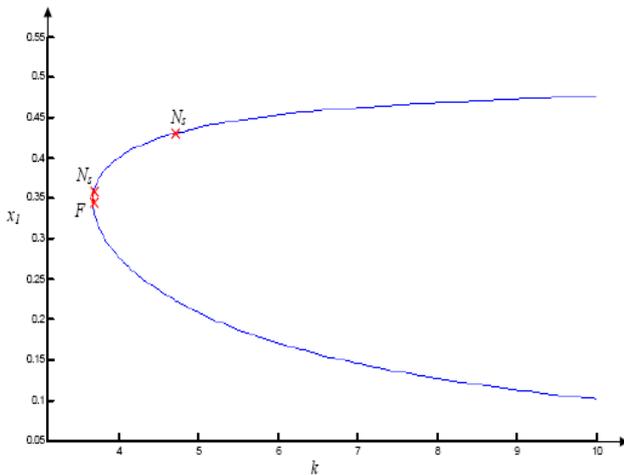

Figure 9. Limit point and Neutral Saddle Points

The three singularities detected in this section are to be used as starting points to trace the bifurcation curves in a two parameter plane chosen here as (k, T_L)-plane depending mainly on the rotor resistor and the rotor time constant.

4.3. Cusp Point and Bogdanov-takens Bifurcation

The continuation of the limit point (LP) detected in previous section leads to trace a saddle-node bifurcation curve shown in Fig.10. Such curve includes two branches joining in a codimension two bifurcation points, namely cuspidal point (CP) having the following phase space coordinates: $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.5, -0.288675, 0.0, 0.57735)$.

Besides, such curve presents another codimension two bifurcation in $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.265302, -0.18176, 0.0, 0.890591)$,

having two eigenvalues equals to zero and known as Bogdanov-Taken bifurcation (BT). Then, using the Hopf bifurcation points, met in the same continuation path of the equilibrium point as the limit point (LP) in previous section, we obtain the Hopf bifurcation curve in (k, TL) - plane, which is seemingly enclosed in the saddle-node bifurcation curve as in Figure 11.
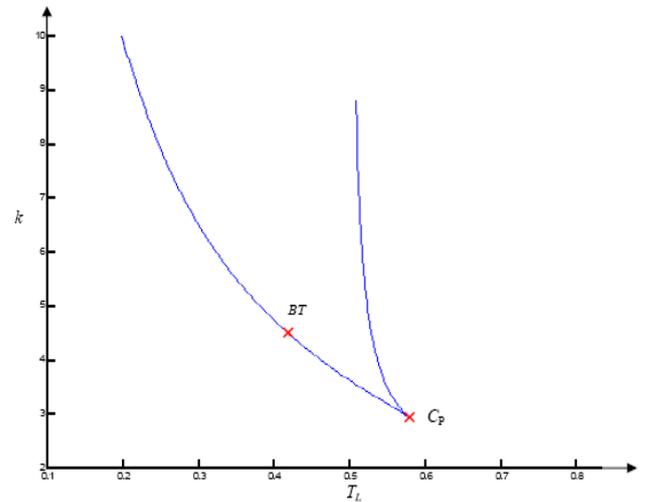

Figure 10. Saddle-node bifurcation including a Cusp point and a Bogdanov-Taken bifurcation
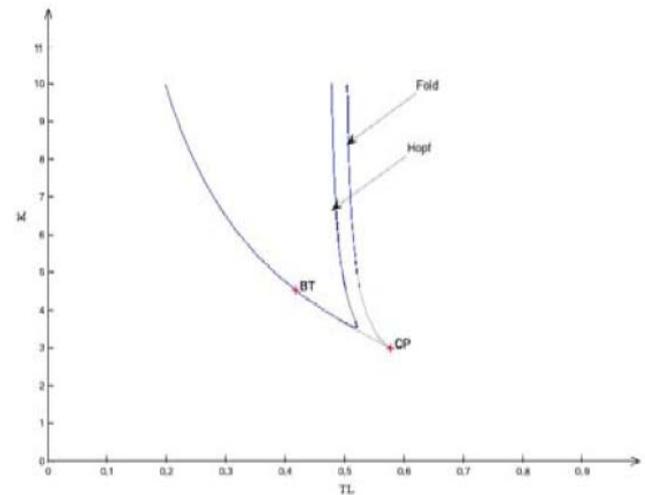

Figure 11. Fold and Hopf bifurcation curves in (k, TL)-plane

The left branches of the two different bifurcation curves seem to be merged together but are not really. Performing several 'zooms' of this part permits to realize that there is no intersections between such bifurcation curves, therefore the Hopf bifurcation curve is completely contained inside the quasi-lip structure. Varying rotor time constant TL from 0 to 1.1, the continuation of an equilibrium point $(x_{10}^*, x_{20}^*, x_{30}^*, x_{40}^*) = (0.7236, -0.3618, 0, 0.191)$ computed for the parameters $k = 4$, $k_p = .4$, $k_i = 1$ and $T_L = 0.5$ is illustrated in Figure 12. Such curve includes two limit points F_1 and F_2 and two Hopf bifurcation points H_1 and H_2, these bifurcation points lead to the same results obtained above and illustrate the fact that the two saddle-node bifurcation curves are the junction of three different sheets, and the Hopf bifurcation is located on the inner sheet between the upper and the lower ones.
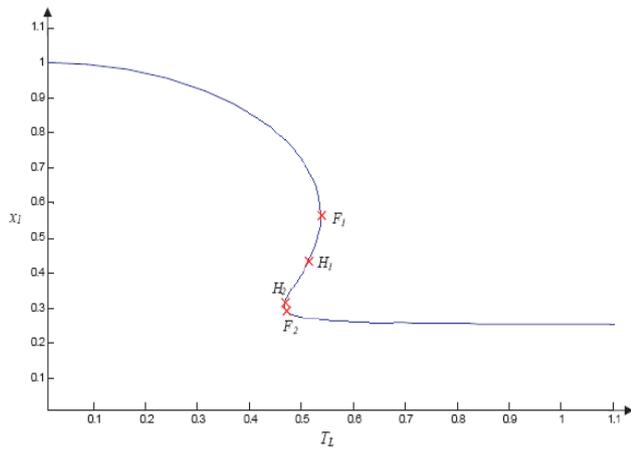
Figure 12. Limit points and Hopf bifurcation

4.4. Hopf Bifurcation of PI Controller Parameters

Aiming to study the impact of the PI controller parameters (k_i and k_p) on the bifurcation structure in the (k, T_L)-parameter plane, a set of Hopf bifurcation curves are traced for a small range of load torque values T_L in Figure 13 for different values of k_p. These bifurcation curves were obtained for ($k_i = 1$).
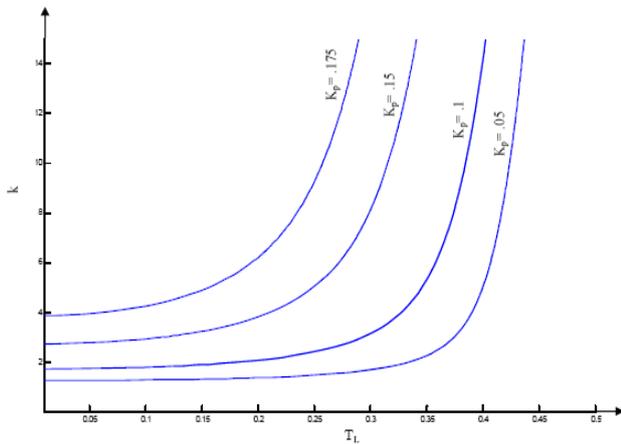

Figure 13. Values of k corresponding to a Hopf bifurcation vs. *TL* for different values of *kp*

For the same range of load torque, a second set of bifurcation curves obtained for fixed ($kp = 0.1$) and for certain values of (ki) is given in Figure 14.
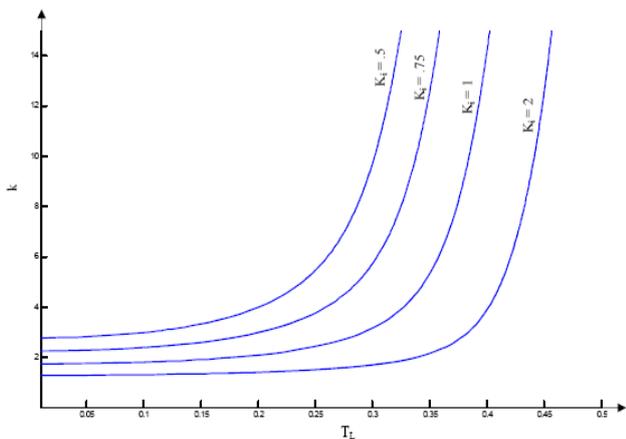

Figure 14. Values of k corresponding to a Hopf bifurcation vs. *TL* for different values of *ki*

For a larger range of load torque values, another set of Hopf bifurcation curves with different shapes presenting an extremum computed for different values of kp in the same parameter plane (k, T_L), see Figure 15.
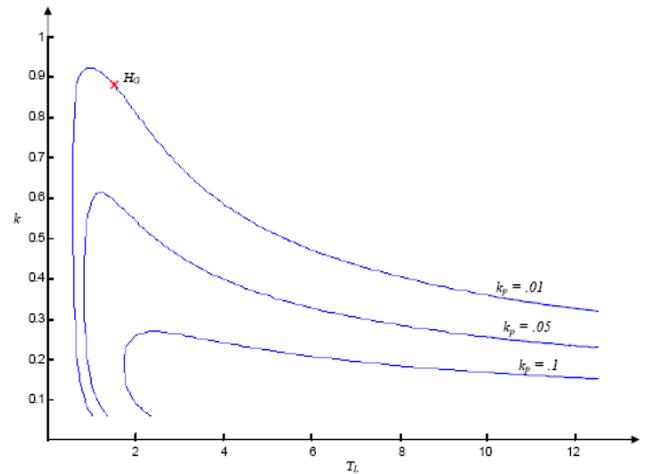

Figure 15. Values of k corresponding to a Hopf bifurcation vs. T_L for different values of kp and for larger values of T_L

4.5. Hopf Bifurcation Curves for Different Value of T_L

In Figure 16, we trace the Hopf bifurcation curve in (kp, ki)- plane for T_L=2.5-5.5-7.5 and 10. In both cases of T_L=7.5 and T_L=10 a codimension two bifurcation point, namely a Generalized Hopf bifurcation is detected.
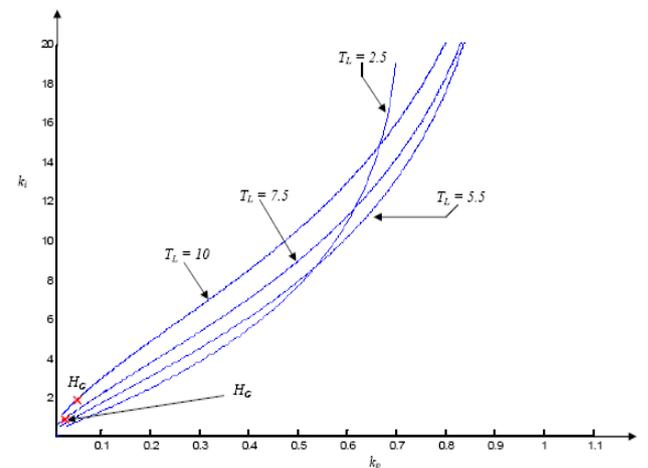

Figure 16. Generalized Hopf bifurcation curves

Such bifurcation is a control bifurcation because it depends on PI controller parameters k_p and k_i.

5. Transition Hopf Bifurcation-Chaotic Behavior

The variation of parameter *ki* from 0.21 to 120 shows the appearance of equilibrium points which undergoes a Hopf bifurcation in the k_i-interval [0.22, 0.3] giving rise to a limit cycle. The phase portraits of the limit cycles present a cuspidal point around which an oscillating part of the trajectory is as important as k_i increases.
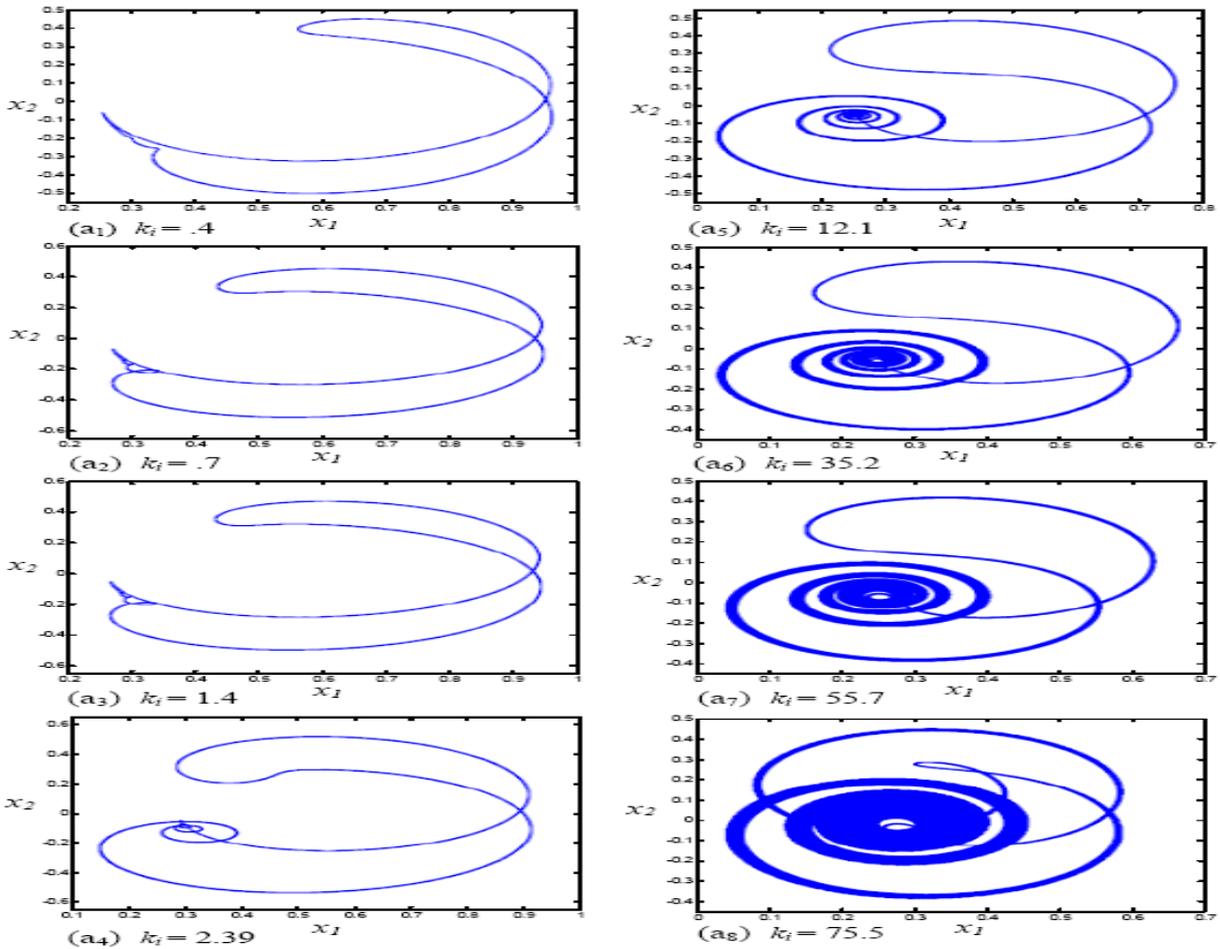
Figure 17. Transition Hopf bifurcation-chaotic behaviour

We recall that according to previous studies [15,20], this phenomenon was related to the important role of higher harmonics whose amplitudes become as more important as the number of modulations is great. But the main result to be emphasized here is that the increasing oscillations around the phase trajectory cuspidal point lead to a chaotic behaviour as shown in Figure 17. The nature of the possible bifurcation scenarios that may occur in k_i-interval [0.3, 75.5], and which exhibit the qualitative change seemingly spectral change of behaviours needs to be deeply investigated.

The Figure 17 was obtained for the same initial conditions set $(x_{10}, x_{20}, x_{30}, x_{40}) = (0.07, 0.152, 0.953, 0.71)$, for the parameter values $k = 4$, $k_p = .01$, $T_L = .5$ and for different values of k_i.

6. Conclusion

Through bifurcation analysis we put into evidence the appearance of equilibrium points, limit cycles and chaotic behaviours in AC drives with respect to parameters mismatches. Moreover, we compute, in phase plane, the stability margins of phase singularities (equilibrium points, limit cycles) and, consequently, illustrate the multistability property. Investigation in different parameter planes had led to find out bifurcation structures related to saddle-node, Fold and Hopf bifurcations in the IFOC of induction motors. It can be observed that the variations of rotor parameters (resistor, time constant) and the control parameters of the PI controller (k_i and k_p) can lead to loss of stability. Such results provide useful guidelines for the setting of tunable parameters keeping all possible instabilities far enough from a practical operating cell of the parameter space. One transition mode to chaotic states via Hopf bifurcation is presented in this paper, other modes, specifically, through increasing of higher harmonics presence in state variable spectra remain an open issue and should be let to further researches. Results and comments given in this paper permit to widen the understanding of the mechanism of certain types of singularities and the stability domain boundaries either in phase space or in parameter space and to demonstrate the suitability of bifurcation theory to solve stability problems in electric machines.

Nomenclature

L_r	Rotor inductance.
R_r	Rotor resistance.
L_m	Mutual inductance.
J	Moment of inertia.
n_p	Pole pair number.
$\tau_r = L_r / R_r$	Rotor flux time constant.
k_p, k_i	The PI controller gains.
τ_e	Estimate of the rotor flux time constant.

u_2^0	Constant reference for flux magnitude
T_L	Load Torque.
LP	Limit point or saddle-node bifurcation.
BT	Bogdanov-Taken bifurcation.
H	Hopf bifurcation.
CP	Cusp point.
LC	Limit cycle.
GH	Generalized Hopf bifurcation.

Acknowledgement

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